Name: ______________________________ Date: ______________________

Average Rate of Change for Exponentials Functions

1: What is the average rate of change of the function $g(x) = -2x + 6$

A.) Over the interval [2, 6]?

B.) Over the interval [5, 7]?

C.) Do you think it is true that g(x) will have a constant average rate of change over *any* interval? Why or why not?

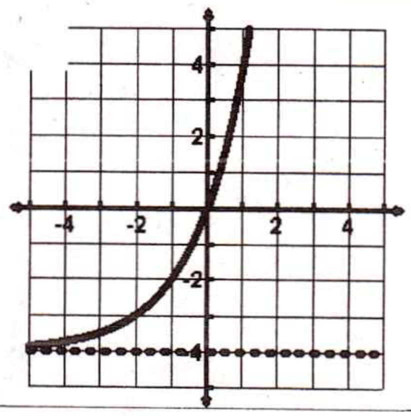
2: What is the average rate of change of the function $f(x) = 2^x$

A.) Over the interval [1, 4]?

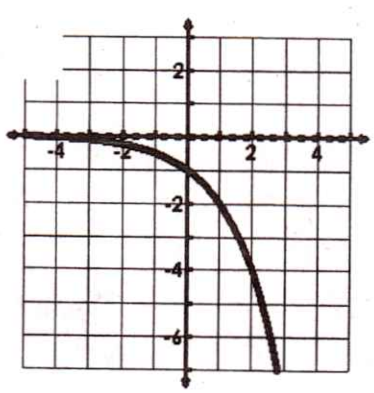
B.) Over the interval [3, 5]?

C.) Do you think it is true that $f(x)$ will have a constant average rate of change over any interval? Why or why not?

3. ROC from [-1, 1]: ____________
4. ROC from [-2, 0]: ____________
5. ROC from [0, 1]: ____________



6. ROC from [0, 1]: ____________
7. ROC from [1, 2]: ____________
8. ROC from [0, 2]: ____________



Given a table, find the rate of change for each interval.

x	y
-3	4
-2	1
-1	0
0	1
1	4
2	9
3	16

7.) [0, 3]

8.) [-2, 1]

9.) [-3, -1]

x	y
-4	.125
-3	.25
-2	.5
-1	1
0	2
1	4
2	8

10.) [-2, 2]

11.) [-4, 1]

12.) [-3, 0]

Plug the function into the table of your calculator. Find the 2 ordered pairs. Then find the Average Rate of Change (slope).

13. $h(x) = 0.5^x$ over the interval $[-1, 0]$.

14.) $g(x) = 1.5^x$ over the interval $[-1, 0]$.

13. $h(x) = 3(2)^{x+1}$ over the interval $[-1, 0]$.

14.) $g(x) = -\left(\frac{1}{4}\right)^x + 2$ over the interval $[-3, -1]$.

Day 5 – Applications of Exponential Functions – Growth/Decay

Review of Percentages: In order to be successful at creating exponential growth and decay functions, it is important you know how to convert a percentage to a decimal. Remember percentages are always out of 100.

Option 1: ______________________ **Option 2:** ______________________

25% = ______ 6.5% = ______ 2% = ______ 10% = ______ 3.05% = ______

Exponential Growth and Decay

As you have already begun to notice, we have been discussing growth and decay quite a bit with exponential functions. You already know how to identify a growth and decay function just from looking at the equation. In case you have forgotten, here are a few practice problems:

A. $y = 8(4)^x$ B. $f(x) = 2(5/7)^x$ C. $h(x) = 0.2(1.4)^x$ D. $y = \frac{3}{4}(0.99)^x$ E. $y = \frac{1}{2}(1.01)^x$

Exponential Growth is where a quantity increases over time where **exponential decay** is where a quantity decreases over time. When we discuss exponential growth and decay, we are going to use a slightly different equation than $y = ab^x$. When you simplify your equation, it will look like $y = ab^x$, but to begin, you will use the following formulas:

Exponential Growth

$y = a(1 + r)^t$

where $a>0$

y = final amount
a = initial amount
r = growth rate (express as decimal)
t = time

(1 + r) represents the growth factor

Exponential Decay

$y = a(1 - r)^t$

where $a>0$

y = final amount
a = initial amount
r = decay rate (express as decimal)
t = time

(1 – r) represents the decay factor

Finding Growth and Decay Rates

Example 1: Identify the following equations as growth or decay. Then identify the initial amount, growth/decay factor, and the growth/decay percent.

a. $y = 3.5(1.03)^t$

Growth/Decay: _______

Initial Amount: ________

Growth/Decay Factor: _________

Growth/Decay Percent: ________

b. $f(t) = 10{,}000(0.95)^t$

Growth/Decay: _______

Initial Amount: ________

Growth/Decay Factor: _________

Growth/Decay Percent: ________

c. $g(t) = 400(0.925)^t$

Growth/Decay: _______

Initial Amount: ________

Growth/Decay Factor: _________

Growth/Decay Percent: ________

d. $y = 2{,}500(1.2)^t$

Growth/Decay: _______

Initial Amount: ________

Growth/Decay Factor: _________

Growth/Decay Percent: ________

Growth and Decay Word Problems

Example 2: The original value of a painting is $1400 and the value increases by 9% each year. Write an exponential growth function to model this situation. Then find the value of the painting in 25 years.

Growth or Decay: _________________________

Starting value (a): __________

Rate (as a decimal): _______

Function: _________________________

Example 3: The population of a town is decreasing at a rate of 1% per year. In 2000, there were 1300 people. Write an exponential decay function to model this situation. Then find the population in 2008.

Growth or Decay: ____________________

Starting value (a): __________

Rate (as a decimal): _______

Function: ____________________

Example 4: The cost of tuition at a college is $12,000 and is increasing at a rate of 6% per year. Find the cost of tuition after 4 years.

Growth or Decay: ____________________

Starting value (a): __________

Rate (as a decimal): _______

Function: ____________________

Example 5: The value of a car is $18,000 and is depreciating at a rate of 12% per year. How much will your car be worth after 10 years?

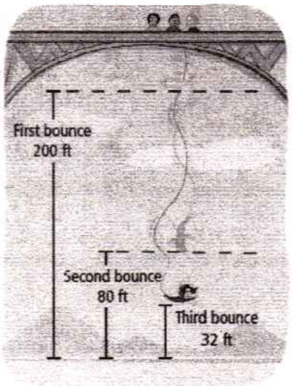
Growth or Decay: ____________________

Starting value (a): __________

Rate (as a decimal): _______

Function: ____________________

Example 6: A bungee jumper jumps from a bridge that is 500 feet high. The diagram shows the bungee jumper's height above the ground at the top of each bounce. What is the bungee jumper's height at the top of the 5th bounce?



Growth or Decay: ____________________

Starting Value: __________

Rate (as a decimal): _______

Function: ____________________

Summary of Exponential Word Problems

Creating a Growth Function Given a Percentage Rate

The number of chickens in the farm of Sunny House is currently 2,400. The farm grows at an annual rate of 15%. How many chickens will be there in 7 years?

Growth: $y = a(1 + r)^t$
Increase
Grow
Appreciate
Gains

Creating a Decay Function Given a Percentage Rate

A limousine costs $75,000 new but depreciates at a rate of 23% per year. What is the value of the limousine after five years?

Decay: $y = a(1 - r)^t$
Decreases
Decays
Depreciates
Loses

Creating an Exponential Function without Being Given a Percentage Rate

A 5th grade class is raising meal worms for an experiment. They start with 10 meal worms. The population triples every hour. How many meal worms does the class have after 12 hours?

Special Key Words
Doubles ($b = 2$)
Triples ($b = 3$)
Half ($b = \frac{1}{2}$)
These values replace ($1 \pm r$)

Creating an Exponential Function Given a Pattern

A population of bees is decreasing. The population in a particular region this year is 1250. After year 1, it is estimated that the population will be 1000. After 2 years, it is estimated that the population will be 800. What will the population be in 6 years?

Without a Given Rate:

$y = a \cdot b^x$

a: starting amount

b: multiplier (constant ratio)

Determine if pattern is growth or decay

Working Backwards to Find the Time (Use Table)

The population of a small town has established a growth rate of 3% per year. If the current population is 2000, and the growth rate remains steady, how many years will it take for the population to first go over 3000?

Working Backwards

-Create your equation

-Input into y =

-Use table to find given y-value and its corresponding x-value (t)

Day 6 – Applications of Exponential Functions – Compound Interest

As you get older, you will come to learn a great deal about investing your money...savings accounts, stock market, mutual funds, bonds, etc. Today, we are going to learn about compound interest, which is a form of saving and earning money by letting it sit in an account over time. **Compound Interest** is interest earned or paid on both the principal and previously earned interest. In middle school, you learned about **simple interest**, which is interest that is only earned on the principal. It's formula is $I = Prt$, where P represents principal, r represents rate, t represents time, and I represents interest.

Compound Interest

$$A = P(1+\frac{r}{n})^{nt}$$

A = balance after *t* years
P = Principal (original amount)
r = interest rate (as a decimal)
n = number of times interest is compounded per year
t = time (in years)

Example 1: Write a compound interest function that models an investment of $1000 at a rate of 3% compounded quarterly. Then find the balance after 5 years.

P = ________

r = ________

n = ________

t = ________

Example 2: Write a compound interest function that models an investment of $18,000 at a rate of 4.5% compounded annually. Then find the balance after 6 years.

P = ________

r = ________

n = ________

t = ________

Example 3: Write a compound interest function that models an investment of $4,000 at a rate of 2.5% compounded monthly. Then find the balance after 10 years.

P = ________

r = ________

n = ________

t = ________