

Day 6: Solving by Completing the Square

**Review: Solving Quadratics**

1)  $2x^2 - 7x + 8 = 2$

$$\begin{array}{r} 2x^2 - 7x + 8 = 2 \\ -2 \quad -2 \\ \hline 2x^2 - 7x + 6 = 0 \end{array}$$

$\frac{-3}{2}$   $\frac{-3}{2}$   $\frac{-4}{2} \rightarrow -2$

$$\begin{array}{l} 2x-3=0 \quad X-2=0 \\ +3+3 \quad +2+2 \\ \hline 2x=3 \quad X=2 \end{array}$$

$(2x-3)(x-2) = 0$

$x = \frac{3}{2}$

2)  $12x^2 - 3x = 0$

$$\frac{12x^2}{3x} - \frac{3x}{3x} = 0$$

$3x(4x-1) = 0$

$$\begin{array}{l} 3x=0 \quad 4x-1=0 \\ \frac{3x}{3} \quad \frac{4x-1}{+1+1} \\ \hline x=0 \quad 4x=1 \\ \frac{4x}{4} \quad \frac{1}{4} \\ \hline x=0 \quad x=\frac{1}{4} \end{array}$$

3)  $-5x^2 + 2 = -48$

$$\begin{array}{r} -5x^2 + 2 = -48 \\ -2 \quad -2 \\ \hline -5x^2 = -50 \\ \frac{-5x^2}{-5} = \frac{-50}{-5} \end{array}$$

$x^2 = 10$

$x = \pm \sqrt{10}$

**Solving by Completing the Square**

Solve the following quadratic equation by completing the square.

$x^2 + 8x - 20 = 0$

Steps	Example
1) Gather $x^2$ and $x$ term on the left side and all constants on the right side of the equals sign.	$\begin{array}{r} x^2 + 8x - 20 = 0 \\ +20 \quad +20 \\ \hline x^2 + 8x = 20 \end{array}$
2) If $a$ is not 1, divide all terms by $a$ . (on both sides of the equals sign)	$x^2 + 8x = 20$ <p><i>*a=1 here; skip step</i></p>
3) Divide $b$ by 2, and add that number to both the left and right sides of the equation. <i>square it!</i>	$b=8 \quad \left(\frac{8}{2}\right)^2 \rightarrow (4)^2 \rightarrow 16$ $x^2 + 8x + 16 = 20 + 16$
4) Rewrite the left side as a binomial squared. $(x + \frac{b}{2})^2$ and simplify the right side of the equation.	$(x+4)^2 = 36$
5) Take the square root of each side of the equation.	$\sqrt{(x+4)^2} = \sqrt{36}$ $x+4 = \pm 6$
6) Separate into two equations, one positive and one negative, and solve for $x$ .	$\begin{array}{r} x+4=6 \\ -4 \quad -4 \\ \hline x=2 \end{array} \qquad \begin{array}{r} x+4=-6 \\ -4 \quad -4 \\ \hline x=-10 \end{array}$

**Practice: Solve by Completing the Square**

$$1) x^2 + 4x - 28 = -7$$

$$\begin{array}{r} x^2 + 4x - 28 = -7 \\ +28 \quad +28 \\ \hline x^2 + 4x = 21 \end{array}$$

$$\left(\frac{4}{2}\right)^2 \rightarrow (2)^2 \rightarrow 4$$

$$(x+2)^2 = 25$$

$$x+2 = \pm 5$$

$$\begin{array}{r} x+2=5 \\ -2 \quad -2 \end{array} \quad \begin{array}{r} x+2=-5 \\ -2 \quad -2 \end{array}$$

$$\boxed{x=3} \quad \boxed{x=-7}$$

$$3) 3x^2 - 6x - 14 = -5$$

$$\begin{array}{r} 3x^2 - 6x = 9 \\ \frac{3}{3} \quad \frac{-6}{3} \quad \frac{9}{3} \end{array}$$

$$x^2 - 2x = 3$$

$$\left(\frac{-2}{1}\right)^2 \rightarrow (-1)^2 \rightarrow 1$$

$$(x-1)^2 = 4$$

$$x-1 = \pm 2$$

$$\begin{array}{r} x-1=2 \\ +1 \quad +1 \end{array} \quad \begin{array}{r} x-1=-2 \\ +1 \quad +1 \end{array}$$

$$\boxed{x=3} \quad \boxed{x=-1}$$

$$5) x^2 - 6x - 72 = 0$$

$$x^2 - 6x = 72$$

$$\left(\frac{-6}{2}\right)^2 \rightarrow (-3)^2 \rightarrow 9$$

$$(x-3)^2 = 81$$

$$(x-3)^2 = \pm 9$$

$$\begin{array}{r} x-3=9 \\ +3 \quad +3 \end{array} \quad \begin{array}{r} x-3=-9 \\ +3 \quad +3 \end{array}$$

$$\boxed{x=12} \quad \boxed{x=-6}$$

$$2) x^2 - 10x - 46 = 10$$

$$x^2 - 10x = 56$$

$$\left(\frac{-10}{2}\right)^2 \rightarrow (-5)^2 \rightarrow 25$$

$$(x-5)^2 = 81$$

$$x-5 = \pm 9$$

$$\begin{array}{r} x-5=9 \\ +5 \quad +5 \end{array} \quad \begin{array}{r} x-5=-9 \\ +5 \quad +5 \end{array}$$

$$\boxed{x=14} \quad \boxed{x=-4}$$

$$4) 5x^2 + 10x - 18 = -3$$

$$5x^2 + 10x = 15$$

$$x^2 + 2x = 3$$

$$\left(\frac{2}{2}\right)^2 \rightarrow (1)^2 \rightarrow 1$$

$$(x+1)^2 = 4$$

$$x+1 = \pm 2$$

$$\begin{array}{r} x+1=2 \\ -1 \quad -1 \end{array} \quad \begin{array}{r} x+1=-2 \\ -1 \quad -1 \end{array}$$

$$\boxed{x=1} \quad \boxed{x=-3}$$

$$6) x^2 + 80 = 18x$$

$$x^2 - 18x = -80$$

$$\left(\frac{-18}{2}\right)^2 \rightarrow (-9)^2 \rightarrow 81$$

$$(x-9)^2 = 1$$

$$x-9 = \pm 1$$

$$\begin{array}{r} x-9=1 \\ +9 \quad +9 \end{array} \quad \begin{array}{r} x-9=-1 \\ +9 \quad +9 \end{array}$$

$$\boxed{x=10} \quad \boxed{x=8}$$