1) Describe the transformation(s) on the parent function, $f(x) = x^2$, given in the function $f(x) = -(x + 3)^2$?

    a) Reflect across the x-axis and translate right 3

    b) Reflect across the x-axis and translate left 3

    c) Reflect across the x-axis and translate up 3

    d) Reflect across the x-axis and translate down 3

2) Describe the transformation(s) on the parent function, $f(x) = x^2$, given in the function $f(x) = 2(x - 4)^2$?

    a) Translate up 2 and right 4

    b) Translate up 2 and left 4

    c) Stretch by a factor of 2 and translate right 4

    d) Stretch by a factor of 2 and translate left 4

3) Write a function that represents the parent function, $y = x^2$, after it has been translated 7 down and 3 left.

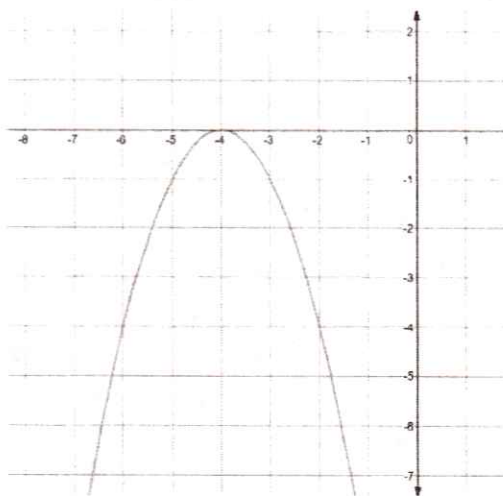   a) $y = (x - 7)^2 + 3$ b) $y = (x - 3)^2 + 7$

    c) $y = (x + 7)^2 - 3$ d) $y = (x + 3)^2 - 7$

4) Which equation models the function graphed below? (Assume that the funciton has not been stretched or shrunk.)

    a) $y = (x + 4)^2 - 1$
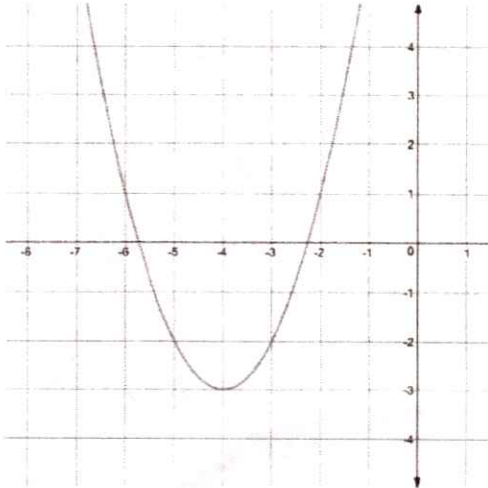
    b) $y = (x - 4)^2 + 1$

    c) $y = -(x + 4)^2$

    d) $y = -(x - 4)^2$

8

5) Identify the vertex of $g(x) = (x + 10)^2 - 3$.

a) $(-3, -10)$ b) $(-3, 10)$ c) $(10, -3)$ d) $(-10, -3)$

6) Identify the vertex of the parabola. Then give the minimum or maximum value of the function.

a) The vertex is (-4, -3) and the minimum is -4.

b) The vertex is (-4, -3) and the maximum is -4.

c) The vertex is (-4, -3) and the maximum is -3.

d) The vertex is (-4, -3) and the minimum is -3.

7) Does the quadratic function $y = -x^2 - 5x - 6$ open up or open down? Explain.

a) Since a is negative, the parabola opens down.

b) Since a is negative, the parabola opens up.

c) Since a is positive, the parabola opens down.

d) Since a is positive, the parabola opens up.

8) Match the equivalent characteristics of a quadratic function which opens down.

_____ x-intercept   a) maximum point

_____ vertex   b) y-coordinate of the vertex

_____ maximum value   c) zero

9) The function $f(x) = -x^2 + 2x$ is graphed below.
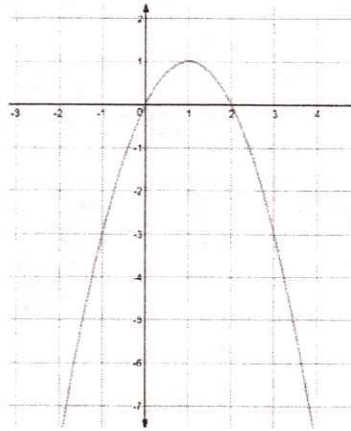
Based on the graph, which statements are true? Select **all** that apply.

    a) The function is **decreasing** on the interval $(-\infty, 1)$.

    b) The function is **decreasing** on the interval $(1, \infty)$.

    c) The function is **increasing** on the interval $(-\infty, 1)$.

    d) The function is **increasing** on the interval $(1, \infty)$.

    e) The function is **positive** on the interval $(-\infty, 0)$.

    f) The function is **negative** on the interval $(-\infty, 0)$.

    g) The function is **positive** on the interval $(0, 2)$.

    h) The function is **negative** on the interval $(0, 2)$.

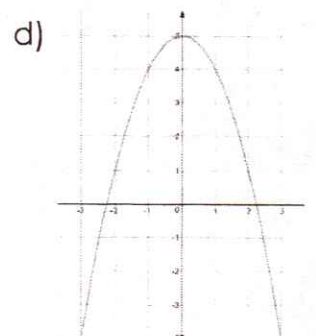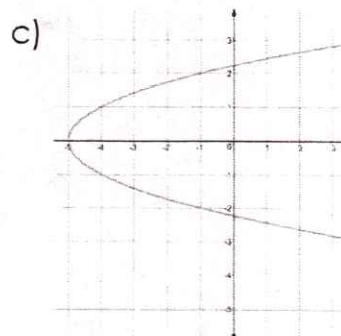10) Which function below has a vertical **shrink**?
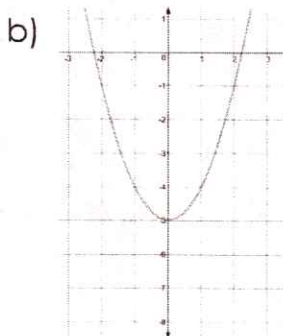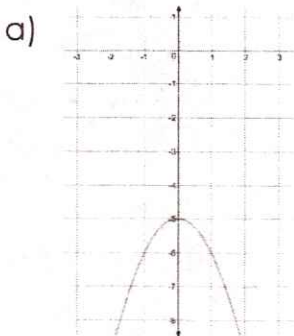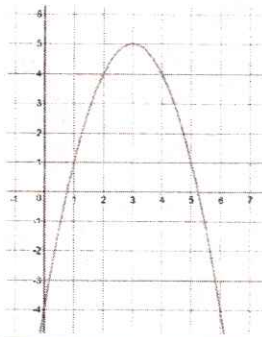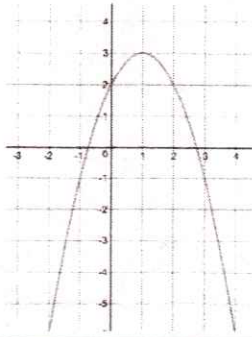
    a) $y = \frac{6}{5}(x + 2)^2$ b) $y = 0.5x^2 + 3$

    c) $y = -\frac{10}{3}x^2 - 4$ d) $y = 4.5(x - 2)^2$

11) Which graph represents the function $y = -x^2 + 5$?

a)   b)   c)   d)

10

12) Match each graph with the correct equation for its axis of symmetry and its maximum OR minimum value.

a) $x = 3$

e) $y = 3$

b) $x = -3$
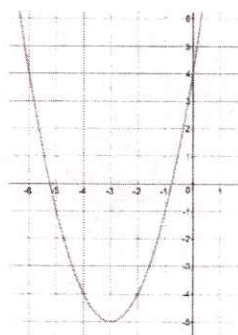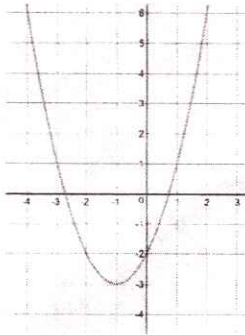
f) $y = -3$

c) $x = 1$

g) $y = 5$

d) $x = -1$

h) $y = -5$

Axis of Symmetry _____

Max / Min (circle one) value _____

Axis of Symmetry _____

Max / Min (circle one) value _____

Axis of Symmetry _____

Max / Min (circle one) value _____

Axis of Symmetry _____

Max / Min (circle one) value _____

13) True / False: Determine whether each statement is true or false for the parabola graphed below.
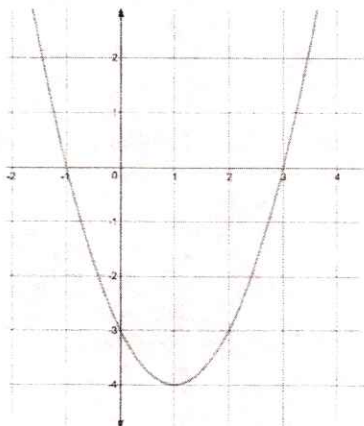
T / F   The equation of the axis of symmetry is $x = 1$.

T / F   The parabola has a maximum point.

T / F   The vertex is $(-1, -4)$.

T / F   The parabola has two x-intercepts.

T / F   The y-intercept is $(0, 0)$.

T / F   The parabola is negative on the interval $(-1, 3)$.

T / F   The zeros of the function are $x = -1$ and $x = 3$.

11

Day 8 – Comparing Vertex and Standard Forms

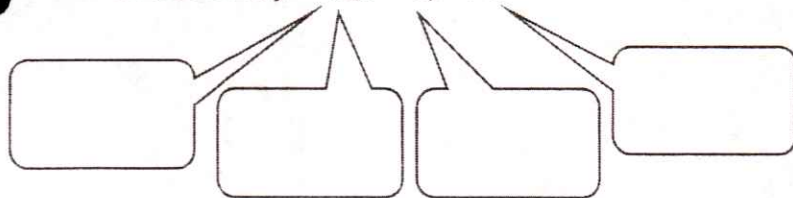Vertex Form: $y = a(x - h)^2 + k$ Standard Form: $y = ax^2 + bx + c$

Which of the following functions opens down? (check all that apply)

- $y = -2(x + 1)^2 + 3$
- $y = -x^2 + 4$
- $y = (x - 4)^2 + 1$
- $y = \frac{1}{2}x^2 + 2x + 8$
- $y = -x^2 - 5x + 6$

Which of the following has a y-intercept of 4? (check all that apply)

- $y = -x^2 + 4$
- $y = x^2 + 4x + 5$
- $y = (x + 4)^2 + 4$
- $y = x^2 + 9x + 4$
- $y = (x - 2)^2 + 4$

Which of the following has a vertical stretch? (check all that apply)

- $y = -2(x + 1)^2 + 3$
- $y = \frac{1}{2}x^2 + 2x + 8$
- $y = 4x^2 + 4$
- $y = x^2 + 4x + 5$
- $y = \frac{5}{2}(x - 4)^2 + 1$

*Can you determine the **y-intercept** of a function simply by looking at the **vertex form** of its equation?

*Can you determine the **vertex** of a function simply by looking at the **standard form** of its equation?

*Can you determine whether a function has a **horizontal shift** simply by looking at the **standard form** of its equation?

Converting from Vertex ⇒Standard Form

Ex 1: Convert to standard form: $y = 2(x + 1)^2 - 4$

1) Expand the $(\ )^2 \Rightarrow (\ )(\ )$, multiply and add x-terms
2) Distribute a, if necessary
3) Combine like terms
DONE!

12

You Try: Convert to STANDARD FORM.

a) $y = (x - 3)^2 + 10$

b) $y = -3(x + 5)^2 - 7$

Direction of opening:

Vertex:

AOS:

y-intercept:

Direction of opening:

Vertex:

AOS:

y-intercept:

Converting from Standard $\Rightarrow$ Vertex Form

Ex 2: Convert to vertex form: $y = x^2 + 4x + 1$

1) Determine values for a, b, and c.

2) Use $x = \dfrac{-b}{2a}$ to find the x-coordinate of the vertex, then substitute x-value into the original equation to find the y-coordinate of the vertex, (h, k).

3) Substitute a, h, and k, into: $y = a(x - h)^2 + k$

DON'T FORGET THAT THE **SIGN WITH h** IN THE EQUATION WILL BE **OPPOSITE!**

You Try: Convert to vertex form: $y = x^2 + 6x + 3$

13

Name_____

1. Convert to standard form: $y = 2(x + 2)^2 - 6$

●

2. What is the extreme of the function $y = -2x^2 + x - 3$, and why?

    a. Minimum, because a is negative.
    b. Minimum, because a is positive.
    c. Maximum, because a is negative.
    d. Maximum, because a is positive.

3. What is the equation of the axis of symmetry for the function $y = x^2 + 6x + 6$.

●

4. Which graph best represents the function $y = x^2 + 6x + 6$.

a.   b.   c.   d.
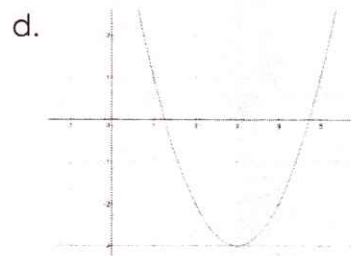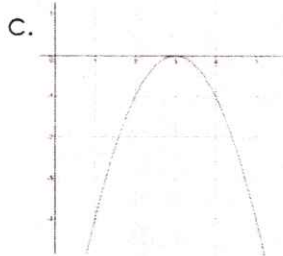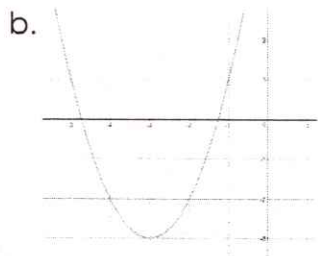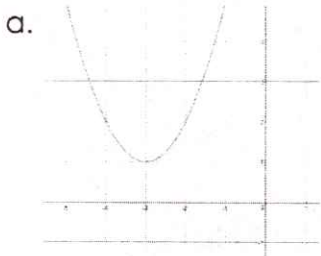
5. What are the coordinates of the vertex of the function $y = x^2 - 8x + 2$?

a. (-8, 2)   b. (8, 2)   c. (4, -14)   d. (-4, 50)

6. What are the coordinates of the vertex of the function $y = (x + 3)^2 - 4$?

● (3, -4)   b. (-3, -4)   d. (-4, 3)   d. (4, -3)

14

Name _____

Sketch a quadratic function with each of the following characteristics:
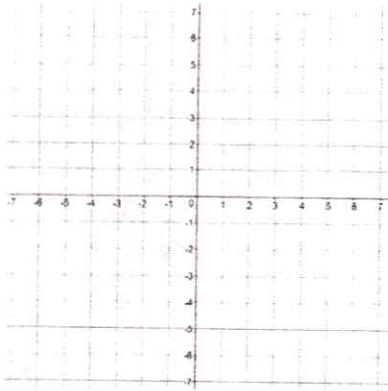
7. Vertex at (-1, 3)	8. Interval of increase (-∞, 2)	9. No x-intercepts
10. Axis of symmetry $x = 1$	**11. y-intercept (0, 4)**	**12. Range (3, ∞)**
13. Positive interval (-2, 2)	14. Negative interval $(-\infty, 1) \cup (5, \infty)$	15. Minimum point (0, -3)
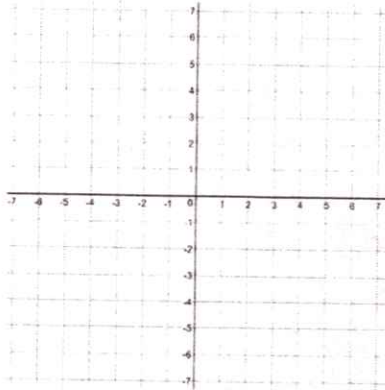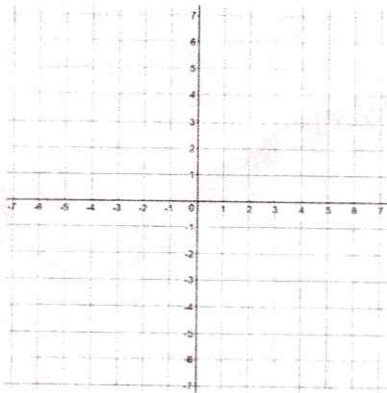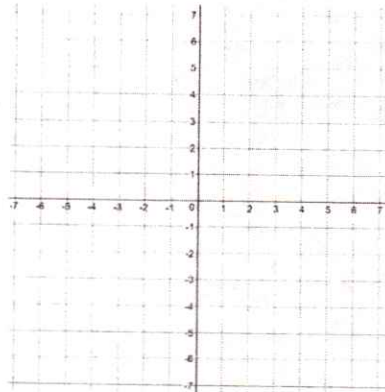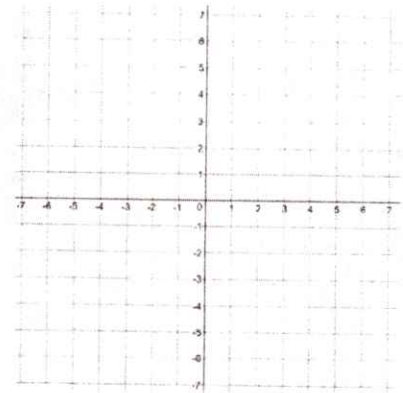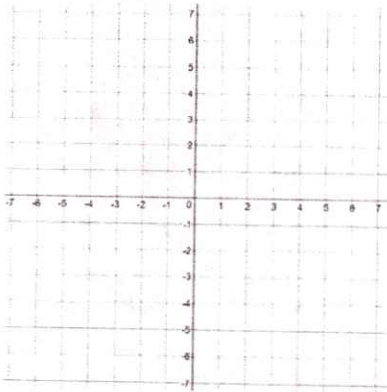
15
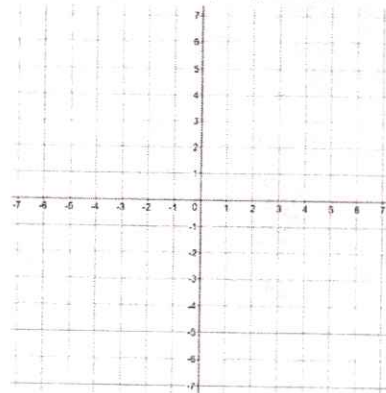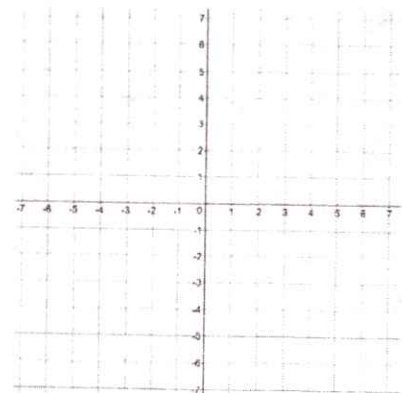
Practice Assignment   Date: _____ Block: _____

Directions: Convert from intercept form to standard form. Then name the y-intercept.

a. $y = (x - 3)(x + 4)$

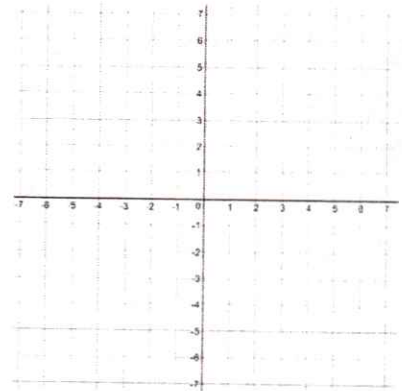b. $y = -(x - 1)(x - 5)$

c. $y = 2(x + 5)(x + 1)$

Form: _____   Form: _____   Form: _____

Y-int: _____   Y-int: _____   Y-int: _____

Directions: Convert from vertex form to standard form. Then name the y-intercept.

a. $y = (x + 5)^2 - 2$

b. $y = -(x - 2)^2 + 6$

c. $y = -3(x - 1)^2 + 4$

Form: _____   Form: _____   Form: _____

Y-int: _____   Y-int: _____   Y-int: _____

Directions: Convert from standard form to intercept form. Then name the x-intercepts.

a. $y = x^2 + 2x - 15$

b. $y = x^2 - 5x - 14$

c. $y = -x^2 + 3x + 4$

rm: _____   Form: _____   Form: _____

X-int: _____   X-int: _____   X-int: _____

16

Directions: Convert from standard form to vertex form. Then name the vertex.

a. $y = x^2 - 10x + 27$ b. $y = -x^2 + 6x - 8$ c. $y = -2x^2 - 24x - 75$

Form: _____   Form: _____   Form: _____

Vertex: _____   Vertex: _____   Vertex: _____

Directions: Convert from intercept form to vertex form. Then name the vertex.

a. $y = (x - 6)(x - 2)$ b. $y = -(x - 5)(x - 3)$ c. $y = \frac{1}{2}(x - 2)(x + 6)$

Form: _____   Form: _____   Form: _____

Vertex: _____   Vertex: _____   Vertex: _____

Day 9 – Different Forms of Quadratics
Practice Assignment

Name: _____

Date: _____ Block: _____

Directions: For the table below, identify each characteristic that can be EASILY determined from looking at the equation (requires no calculations). <u>You will not fill in answers for every box.</u>

Equation	Graph Opens	Vertex	X-Intercepts	Y-Intercept
1. $y = (x + 4)^2 - 5$				
2. $y = -2(x + 3)(x - 2)$				
3. $y = -x^2 + 3$				
4. $y = x^2 + 5x - 14$				
5. $y = -(x + 1)^2$				
6. $y = (x - 7)(x + 5)$				
7. $y = x^2 + 8x + 12$				
8. $y = -2(x - 3)^2 + 1$				

Convert the following equations to the specific form and give the additional characteristics you can determine from the new form.

Equation 1 to standard:	Equation 4 to factored:	Equation 6 to standard:	Equation 7 to vertex:

18

Review: Identify the form each quadratic equation is in. Then graph the equations by calculating the vertex and creating an xy chart.

9. Graph $y = (x - 4)(x + 2)$

Form: _____
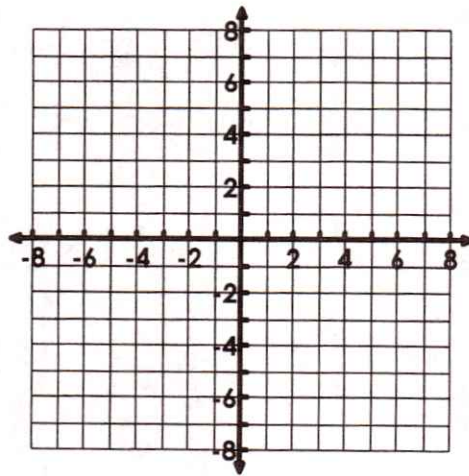
10. Graph $y = x^2 + 4x - 5$

Form: _____
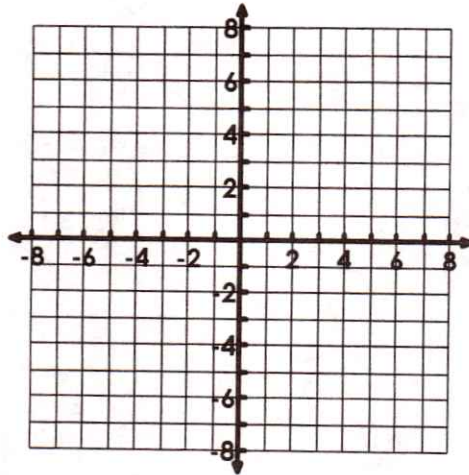
11. Graph $y = -2(x + 3)^2 - 2$

Form: _____

19
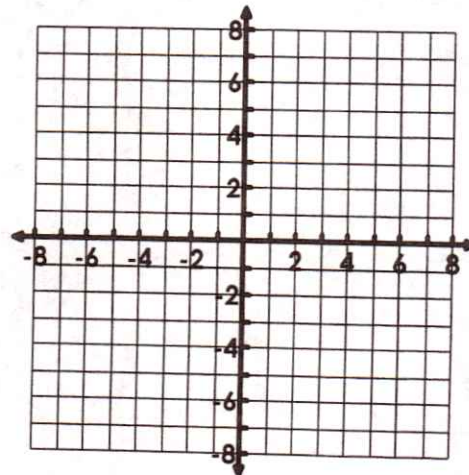
Review of Finding Slope

Practice Assignment

Name: _____

Date: _____ Block: _____

SLOPE is: $\dfrac{\Delta y \text{ (change in y)}}{\Delta x \text{ (change in x)}}$ $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ $\dfrac{\text{Rise}}{\text{Run}}$

1. Calculate the slope and y-intercept from each graph.

A.   Slope = _____   B.   Slope = _____   C.   Slope = _____
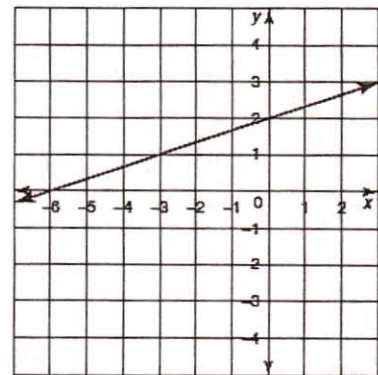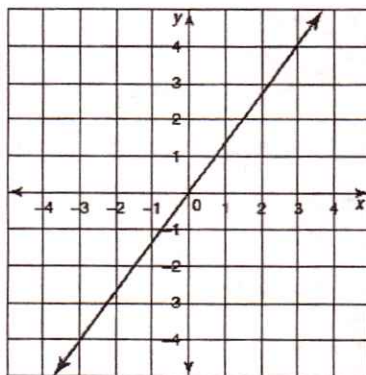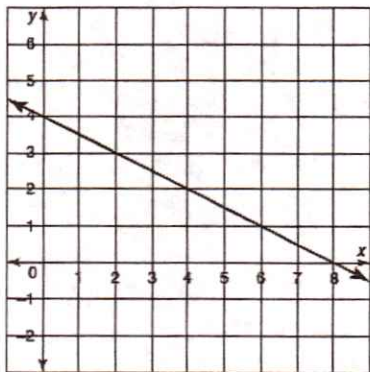
2. Calculate the slope/rate of change from the table.

A.

x	y
−2	8
0	0
2	−8
4	−16

B.

x	y
−10	50
−2	10
4	−20
14	−70

3. Calculate the slope from a set a points.

a. (-1, -24) & (2, 48)   b. (4, -20) & (-10, 50)

20

More Practice with Characteristics: Name the characteristics for each graph given

8.

Domain: _____   Range: _____

Vertex: _____   Axis of Sym._____

Y-Intercept: _____   Zeroes: _____

Extrema: _____   Max/Min Value: _____

Int of Inc: _____   Int of Dec: _____

Positive: _____   Negative: _____

End Behavior: As x → -∞, f(x) → _____.   As x → ∞, f(x) →_____

9.

Domain: _____   Range: _____

Vertex: _____   Axis of Sym._____

Y-Intercept: _____   Zeroes: _____

Extrema: _____   Max/Min Value: _____

Int of Inc: _____   Int of Dec: _____

Positive: _____   Negative: _____

End Behavior: As x → -∞, f(x) → _____.   As x → ∞, f(x) →_____

10. Describe the transformations from the parent function $y = x^2$ to the second graph. Then write the equation of the transformed graph.

21
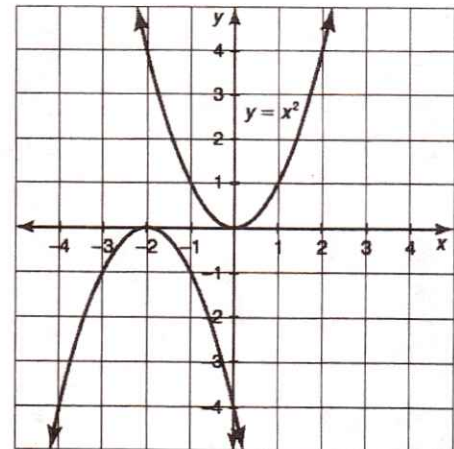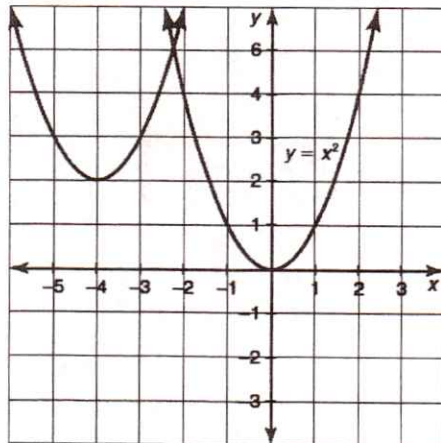
Day 10: Average Rate of Change

Review: Find the slope (average rate of change) for the following problems:

a.

c. (-9, 5) & (-3, 1)

b.

x	y
3	27
5	45
7	63
9	81
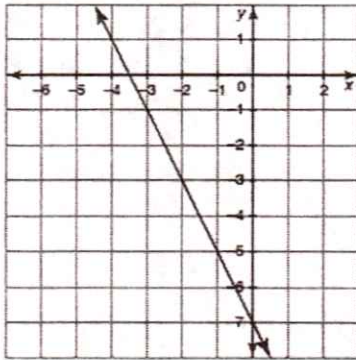
When you calculate the slope of linear function, its slope is ALWAYS _____.

Investigating the "Slope" of a Quadratic Function
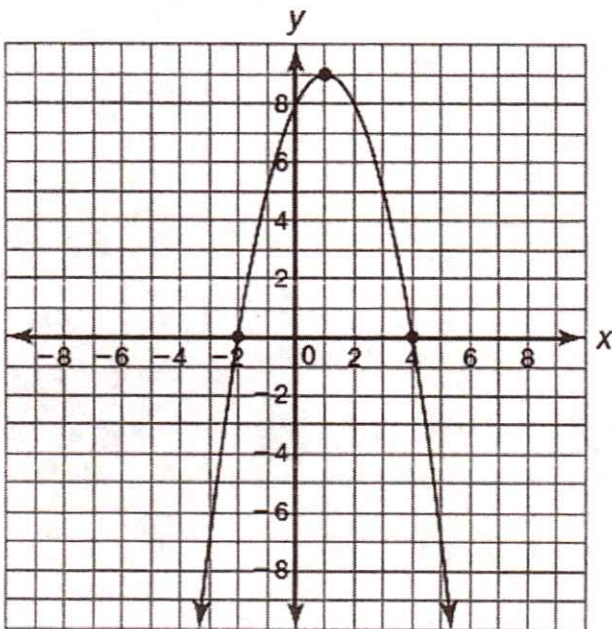
The graph of $y = -x^2 + 2x + 8$ is given. Fill in the table of values on the right. Then determine the slope from one point to the next point.

X	Y
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	

What do you notice about the rate of change as you go from one point to the next?

22

What do you notice if you find the difference of all the slopes?

First versus Second Differences

Quadratic Functions have **constant second differences.** Second differences can be calculated by finding the rate of change with the first differences. Linear functions have **constant first differences.** Since quadratic functions do not have constant first differences, they do not have a slope that remains constant for the entire graph of a parabola.

a. $y = 2x$ _____

x	y	First Differences	Second Differences
-3			
-2			
-1			
0			
1			
2			
3			

b. $y = 2x^2$ _____

x	y	First Differences	Second Differences
-3			
-2			
-1			
0			
1			
2			
3			

Therefore, you are never asked to find the slope of a quadratic function, but rather the **average rate of change** on a given interval. The average rate of change of a quadratic function will be different for each interval you are asked to find, just like in your investigation problem.

Practice: For the problems below, find the average rate of change for the given intervals:

Calculate average rate of change on interval $0 \leq x \leq 2$.

Calculate average rate of change on interval $0 \leq x \leq 3$.
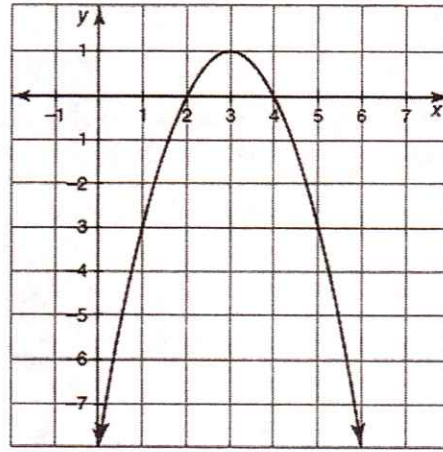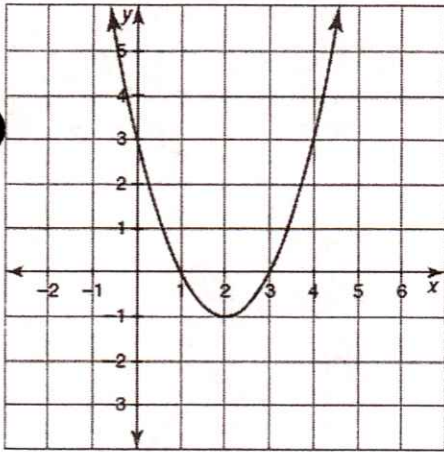
Average Rate of Change without a Graph

If you are asked to calculate the average rate of change on an interval without a graph, you will have to come up with two points to calculate the slope.

You will get your two points by taking the bounds of your interval and substitute those x-values into your equation to find the y-values. Then use the slope formula to calculate the slope.

$$\text{Remember slope is: } \frac{rise}{run} \text{ or } \frac{y_2 - y_1}{x_2 - x_1}$$

Practice: Calculate the average rate of change of the function $y = (x - 4)^2$ on the given intervals:

$1 \leq x \leq 3$ $-2 \leq x \leq 2$

Practice: Calculate the average rate of change of the function $y = x^2 + 4x - 12$ on the given intervals:

$-2 \leq x \leq 4$

$-3 \leq x \leq -6$

Algebra 1
Day 10 – Average Rate of Change

Unit 8: Quadratic Functions

Practice

Name:_____

Practice Assignment

Date:_____ Block:_____

Find the average rate of change for the given intervals:

1. $-3 \leq x \leq 0$
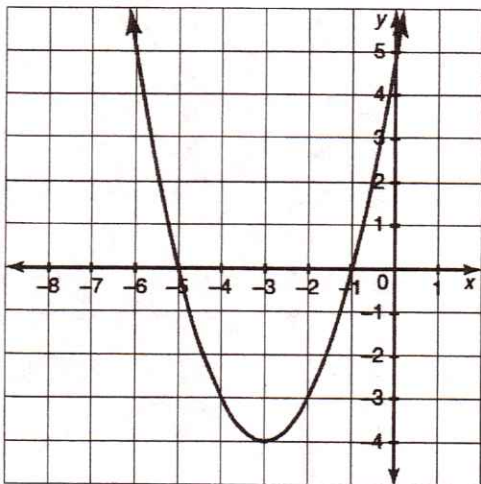
2. $0 \leq x \leq 2$
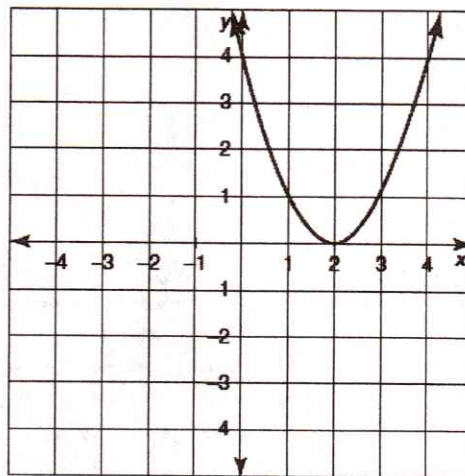
3. $4 \leq x \leq 7$
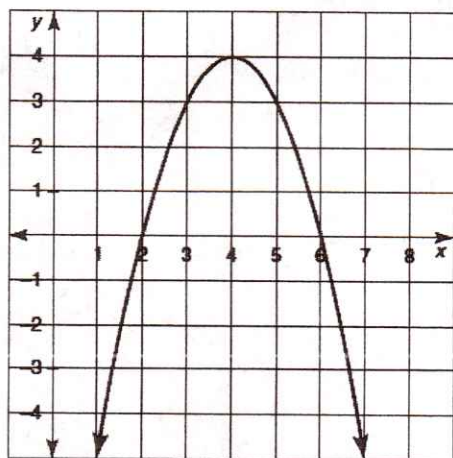
4. $1 \leq x \leq 2$
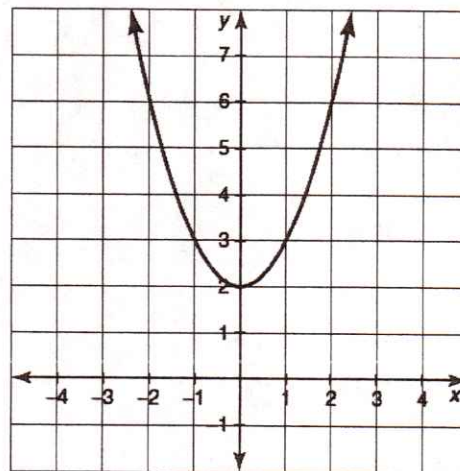
Find the average rate of change for the given equations on the given intervals:

5. $y = x^2 - 4x + 6$; $2 \leq x \leq 4$

6. $y = x^2 - 4x + 1$; $-1 \leq x \leq 2$

7. $y = -x^2 - 6x - 10$; $-7 \leq x \leq -3$

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Day 12 – Comparing Quadratic Functions   Name: _____

Practice Assignment   Date: _____ Block: _____

irections: Answer the following questions to comparing quadratic functions.

1. Which quadratic function has the bigger y-intercept? Explain why.

a. $y = -x^2 + 3x + 8$ b.

X	-4	-3	-2	-1	0	1
y	9	13	19	13	9	7

2. Which quadratic function has the smallest y-intercept? Explain why.

a. $y = x^2 + 4x - 12$ b. $y = (x + 3)(x - 3)$ c. $y = (x + 2)^2 - 13$

3. Which quadratic function has the least minimum value? Explain why.

a.   b.

X	-4	-3	-2	-1	0	1
y	0	-5	-8	-9	-8	-5
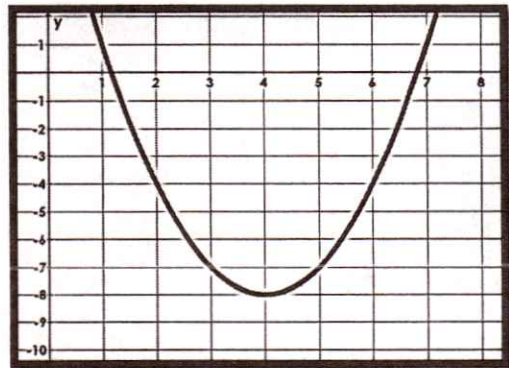
4. Which quadratic function has the greatest minimum value? Explain why.

a. $y = (x + 4)^2 + 2$ b. $y = -(x + 3)(x + 1)$ c.

X	2	3	4	5	6
y	0	-1	0	3	8

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5. Two seagulls dive into the ocean. The given functions represent the height of each seagull above the surface of the ocean as a function of the seagull's horizontal distance from a center buoy. For each set of functions, **determine which bird descends deeper into the ocean.** Support your answer with facts (work).

a.

$$\begin{cases} \text{First Seagull: } f(x) = 3(x-2)^2 - 5 \\ \text{Second Seagull: } g(x) = \{(-8,0),(-6,-4),(-4,0)\} \end{cases}$$

_____

b.

$$\begin{cases} \text{First Seagull: } f(x) = 3x^2 - 12x + 7 \\ \text{Second Seagull: } g(x) = \frac{1}{2}(x+2)^2 - 6 \end{cases}$$

_____

c.

First Seagull: $f(x) = 2x^2 - 8x + 11$

Second Seagull:

x	-3	-1	1	3	5
g(x)	11	6	3	2	3

6. Which function has the lesser maximum value? Why?

A. Parabola with no x-intercepts and a < 0?   **OR**   B. Parabola with two x-intercepts and a < 0?
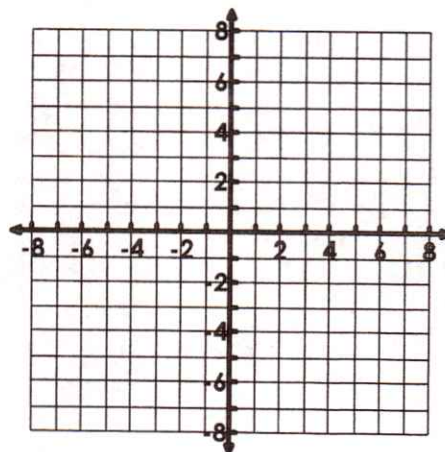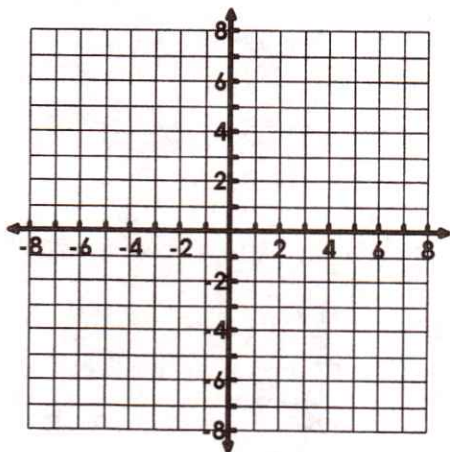
Use the graphs to help explain your answer.

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Quadratic Keywords

vertex

at the vertex:

| **x-value:** When is the object at max height? | **y-value:** How high does the object get? |

y-intercept

How high **off the ground** did the object start?

"How high is the object **after** ____ **seconds?**

Plug given seconds into the graph's equation.

zero

zero

This zero gets thrown out for being too **negative!**

at the positive zero:

| When does the object hit the ground? | How long is the object in the air? |

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Day 11 – Applications of the Vertex

Words that Indicate Finding Vertex	Quadratic Equations
• Minimum/Maximum • Minimize/Maximize • Least/Greatest • Smallest/Largest	Standard Form: $y = ax^2 + bx + c$　　　y-int: $(0, c)$ Vertex Form: $y = a(x – h)^2 + k$　　　vertex: (h, k) Factored Form: $y = a(x – p)(x – q)$　　x-int: $(p, 0)$ & $(q, 0)$ Vertex: $\left(\dfrac{-b}{2a}, f\left(\dfrac{-b}{2a} \right) \right)$

1. The arch of a bridge forms a parabola modeled by the function $y = -0.2(x – 40)^2 + 25$, where x is the horizontal distance (in feet) from the arch's left end and y is the corresponding vertical distance (in feet) from the base of the arch. How tall is the arch?

2. Suppose the flight of a launched bottle rocket can be modeled by the equation $y = -x^2 + 6x$, where y measures the rocket's height above the ground in meters and x represents the rocket's horizontal distance in meters from the launching spot at $x = 0$.

a. How far has the bottle rocket traveled horizontally when it reaches it maximum height? What is the maximum height the bottle rocket reaches?
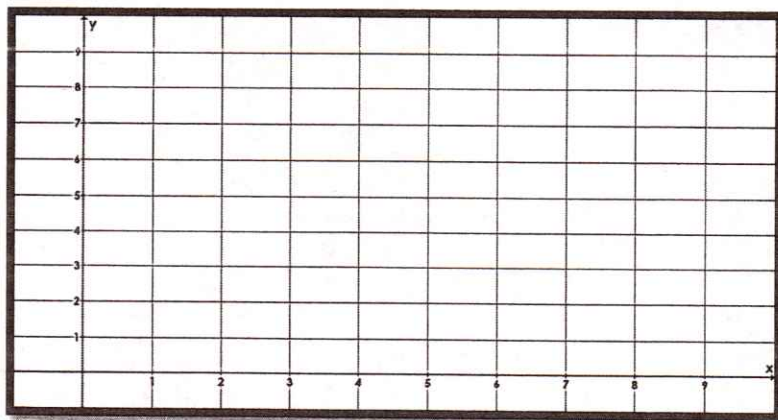
b. How far does the bottle rocket travel in the horizontal direction from launch to landing?

3. A frog is about to hop from the bank of a creek. The path of the jump can be modeled by the equation $h(x) = -x^2 + 4x + 1$, where $h(x)$ is the frog's height above the water and x is the number of seconds since the frog jumped. A fly is cruising at a height of 5 feet above the water. Is it possible for the frog to catch the fly, given the equation of the frog's jump?
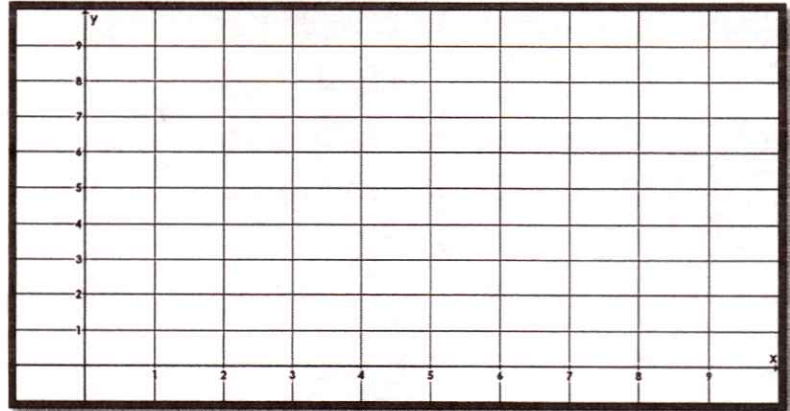
A baker has modeled the monthly operating costs for making wedding cakes by the function $y = 0.5x^2 - 12x + 150$, where y is the total costs in dollars and x is the number of cakes prepared.

a. How many cakes should be prepared each month to yield the minimum operating cost?

b. What is the minimum monthly operating cost?

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5. A street vendor sells about 20 shirts a day when she charges $8 per shirt. If she decreases the price by $1, she sells about 10 more shirts each day.

a. How many shirts does she have to sell to maximize her revenue? What is her maximum revenue?

Price	Number of Shirts Sold	Revenue
$8	20	

b. How much more will she make a day?

c. Write a quadratic function that models the scenario.

6. You run a canoe rental business on a small river in Georgia. You currently charge $12 per hour canoe and average 36 rentals a day. An industry journal says that for every fifty cent increase in rental price, the average business can expect to lose two rentals a day.

a. Use this information to attempt to maximize your income. What should you charge?

Price	Number of Rentals	Revenue
$12	36	

b. Write a quadratic function that models the scenario.

Review – Find the vertex of the following equation: $y = 2x^2 – 4x + 5$.

Directions: Answer the following questions that pertain to using applications of the vertex.

1. The valley between two mountains whose peaks touch the x-axis is $y = 40.4x^2 – 404x$, where x and y are measured in feet. How deep is the valley?

2. A model rocket is launched straight upward. The path of the rocket is modeled by $h = -16t^2 + 200t$, where h represents the height of the rocket and t represents the time in seconds.

    a. What is its maximum height?

   b. Is it still in the air after 8 seconds? Explain why or why not.

    c. Is it still in the air after 14 seconds? Explain why or why not.

3. A model for a company's revenue is $R = -15p^2 + 300p + 12{,}000$, where p is the price in dollars of the company's product. What price will maximize revenue? What will be the maximum revenue?

4. The photo shows the Verranzo-Narrows Bridge in New York, which has the longest span of any suspension bridge in the United States. A suspension of cable of the bridge forms a curve that resembles a parabola. The curve can be modeled with the function **$y = 0.0001432(x - 2130)^2$**, where x and y are measured in feet. The origin of the function's graph is at the base of one of the two towers that support the cable.

a. What is the vertex of the bridge between two towers?

b. How far apart are the towers?
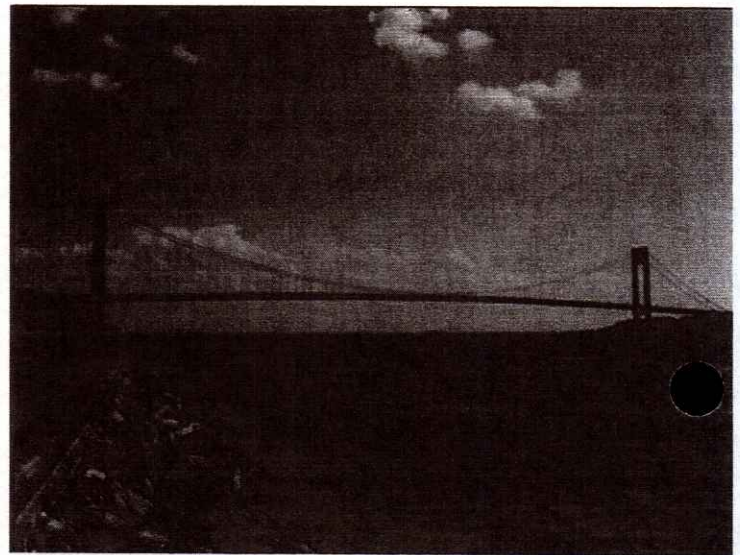
5. A sports store sells about 50 mountain bikes per month at a price of $220 each. For each $20 decrease in price, about 10 more bikes per month are sold. Complete the table below

Price	Bikes	Revenue
220	50	

a. How many prices changes does it take to make a maximum amount of revenue?

b. At what price and how many bikes need to be sold to maximize their revenue?

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