Foundations of Algebra Unit 6

Day 15 – Arithmetic Sequences

For the following patterns, find the next two numbers. Then describe the rule you are applying each time.

Pattern	Rule	Common Difference
a. -4, -2, 0, 2, 4, 6, ...	adding 2	+2
b. -16, -12, -8, -4, 0, 4, ...	adding 4	+4
c. 6.5, 5, 3.5, 2, 0.5, -1, ...	subtracting 1.5	-1.5
d. 12, 18, 24, 30, 36, ...	adding 6	+6
e. 50, 40, 30, 20, 10, ...	subtract 10	-10
f. 11, 9, 7, 5, 3, ...	subtracting 2	-2

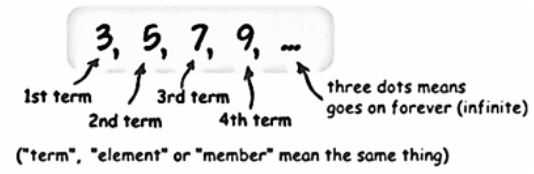
g. What did you notice about your patterns?
they are either adding or subtracting

h. What do you think the "..." means?
the pattern continues forever

Sequences

A **sequence** is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects. A sequence in which you get the next consecutive term by adding or subtracting a constant value is called an **arithmetic sequence**. In other words, we just add or subtract the same value over and over...infinitely. This constant value is called the **common difference**. → # that you +/-

What you may not realize is when it comes to sequences, they are considered linear functions. The position of each term is called the **term number or term position.** We can think of the term number or position as the input (domain) and the actual term in the sequence as the output (range). Instead of using x for the input, we are going to use n and instead of using y for the output, we are going to use a_n.



Pattern A:

Term Number (n)	1	2	3	4	5	6
Term (a_n)	-4	-2	0	2	4	6
	+2	+2	+2	+2	+2	

Pattern D:

Term Number (n)	1	2	3	4	5
Term (a_n)	12	18	24	30	36
	+6	+6	+6	+6	

There are two formulas for arithmetic sequences – the first is called the **Recursive Formula.** The recursive formula allows you to find the next term in a sequence if you know the common difference and any term of the sequence.

$a_1 = \#$ (first term)

$$a_n = a_{n-1} + d$$

a_n: Nth Term; a_{n-1}: Previous Term; d: Common Difference

Finding Terms Using a Recursive Formula

For the following recursive formulas, find the first four terms:

1. $a_1 = 4$ ← 1st term; $a_n = a_{n-1} + 4$ → add 4

4, 8, 12, 16 (+4, +4, +4)

2. $a_1 = -7$ → 1st; $a_n = a_{n-1} - 6$ → −6

−7, −13, −19, −25 (−6, −6, −6)

3. $a_1 = -3.5$; $a_n = a_{n-1} + 9$

−3.5, 5.5, 14.5, 23.5

4. $a_1 = 99$; $a_n = a_{n-1} - 100$

99, −1, −101, −201

5. $a_1 = -17$; $a_n = a_{n-1} + 28$

−17, 11, 39, 67

6. $a_1 = 2$; $a_n = a_{n-1} - 4$

2, −2, −6, −10

Creating a Recursive Rule

* to find the pattern, take the last # and subtract the # before it *

For the following sequences, create a recursive rule:

a. 1, 8, 15,... (+7)

$a_1 = 1$; $a_n = a_{n-1} + 7$

b. 4, 0, -4,... (−4)

$a_1 = 4$; $a_n = a_{n-1} - 4$

c. -5, 3, 11,... (+8)

$a_1 = -5$; $a_n = a_{n-1} + 8$

d. 14, 3, -8,... (−11, −11)

$a_1 = 14$; $a_n = a_{n-1} - 11$

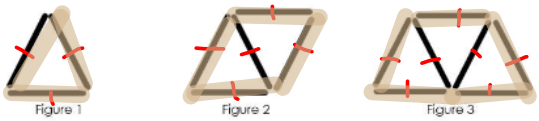
e. 7, 10, 13,... (+3)

$a_1 = 7$; $a_n = a_{n-1} + 3$

f. -6, -13, -20... (−7)

$a_1 = -6$; $a_n = a_{n-1} - 7$

Using Figures to Create Rules



	# of Popsicle Sticks	Perimeter
Figure 1	3	3
Figure 2	5	4
Figure 3	7	5
Figure 4	9	6
Figure 5	11	7
Figure 6	13	8

a. Create a recursive rule for finding the number of Popsicle sticks.

$a_1 = 3$

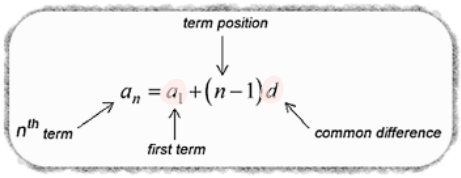
$a_n = a_{n-1} + 2$

b. Create a recursive rule for finding the number of Popsicle sticks in the perimeter.

$a_1 = 3$

$a_n = a_{n-1} + 1$

Explicit Formula:



Why We Have a Formula for Sequences

Take a look at the following pattern: **4, 8, 12, 16, ...** 20, 24, 28, 32

What is the 3rd term? 12 What is the 5th term? 20 What is the 7th term? 28

What is the pattern? adding 4 What is the 1st term? 4

What is the 54th term? ? (You don't want to add 4 over and over 54 times?!?!?!?)

This is why the **Explicit Formula** was created – as long as you know your common difference and 1st term, you can create a rule to describe any arithmetic sequence and use it to find any term you want.

$a_n = a_1 + (n-1)d$ 3, 5, 7, ... (+2)

Creating an Explicit Rule	
1. Write down the Explicit Formula.	$a_n = a_1 + (n-1)d$
2. Substitute the first term in for a_1 and common difference in for d.	$a_n = 3 + (n-1)(2)$
3. Simplify the right side of the equation so that you have an equation that looks very similar to y = mx + b (except it will look more like $a_n = dn + c$).	$a_n = 3 + 2n - 2$ $a_n = 2n + 1$ $a_n = 1 + 2n$
4. To find an nth term, substitute the term number you are wishing to find into n. Find a_{59}. (59th term)	$a_{59} = 2(59) + 1 = 119$

Write an Explicit Rule for the following sequences:

a. 1, 8, 15,... (1st term → a_1, common diff. → d)

$a_1 = 1$

$d = 7$

$a_n = a_1 + (n-1)d$

$a_n = 1 + (n-1)(7)$

$a_n = 1 + 7n - 7$

$a_n = 7n - 6$

b. 4, 0, -4,...

$a_1 = 4$

$d = -4$

$a_n = 4 + (n-1)(-4)$

$a_n = 4 - 4n + 4$

$a_n = -4n + 8$

c. -5, 3, 11,...

$a_1 = -5$

$d = 8$

$a_n = -5 + (n-1)(8)$

$a_n = -5 + 8n - 8$

$a_n = 8n - 13$

For each of the above sequences, find the term before the first term (find a_0). What do you notice about this number and your formula?

a. $a_0 = -6$

-6, 1, 8, 15,... (-7, +7, +7)

b. $a_0 = 8$

$a_n = -4n + 8$

$a_0 = -4(0) + 8$

$a_0 = 0 + 8$

$a_0 = 8$

c. $a_0 = -13$

The 0th term is the constant of the explicit form.

Finding the Nth Term

To find the nth term, particularly when the nth term is quite large, you want to create an Explicit Rule first and then substitute that term number into the rule for n.

For the given sequences, create an explicit rule and then use the rule to find the following terms:

a. 5, 10, 15, 20, ... Find 21st term

$a_1 = 5$, $d = 5$

$a_n = 5 + (n-1)(5)$

$a_n = 5 + 5n - 5$

$a_n = 5n$

$a_{21} = 5(21)$

$a_{21} = 105$

b. 121, 110, 99, 88, ... Find a_{10}

$a_1 = 121$, $d = -11$

$a_n = 121 + (n-1)(-11)$

$a_n = 121 + 11n + 11$

$a_n = 11n + 132$

$a_{10} = 11(10) + 132$

$a_{10} = 242$

c. -30, -22, -14, -6, ... Find a_{30}

$a_1 = -30$, $d = 8$

$a_n = -30 + (n-1)(8)$

$a_n = -30 + 8n - 8$

$a_n = 8n - 38$

$a_{30} = 8(30) - 38$

$a_{30} = 202$

d. 3, 8, 13, 18, ... Find 17th term

$a_1 = 3$, $d = 5$

$a_n = 3 + (n-1)(5)$

$a_n = 3 + 5n - 5$

$a_n = 5n - 2$

$a_{17} = 5(17) - 2$

$a_{17} = 83$

Finding Terms Using an Explicit Rule

For the following sequences, find the first four terms:

a. $a_n = 4 + 3(n-1)$ (a_1, d)

4, 7, 10, 13

+3, +3, +3

b. $a_n = -1(n-1)$ (d, $a_1 = 0$)

0, -1, -2, -3

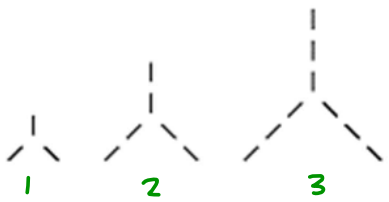
-1, -1, -1

c. $a_n = 9(n-1) + 13$ (d, a_1)

13, 22, 31, 40

+9, +9, +9

Complete the table to the right. Then write an explicit rule to represent the number of dashes.



	# of Dashes
Figure 1	3
Figure 2	6
Figure 3	9
Figure 4	12
Figure 5	15
Figure 6	18

$a_1 = 3$ $d = 3$

$a_n = 3 + (n-1)(3)$

$a_n = 3 + 3n - 3$

$a_n = 3n$